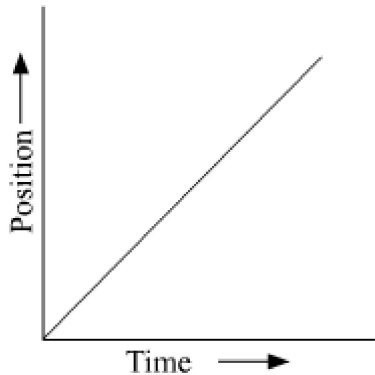


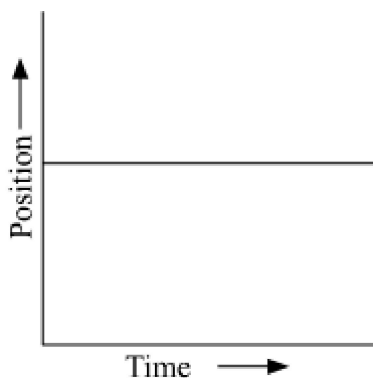
3. Projectile Motion

- An object is at rest when the position of the object does not change with time and with respect to its surroundings.
- An object is in motion when the position of the object changes with time and with respect to its surroundings.
- Rest and motion are relative.
- If the distance covered by an object is much greater than its size during its motion, then the object is considered as point mass object.
- Distance or path length — Total length of the path covered by a body (scalar quantity)
- Displacement — Shortest distance between initial and final positions measured along a particular direction

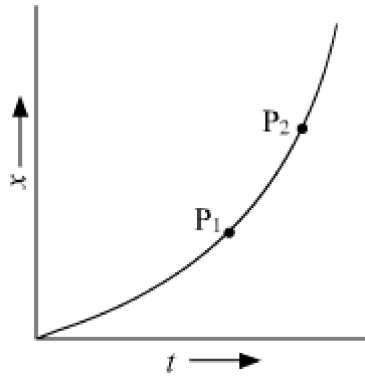
- Uniform motion (object moving with a constant velocity):



- Stationary object (object at rest):



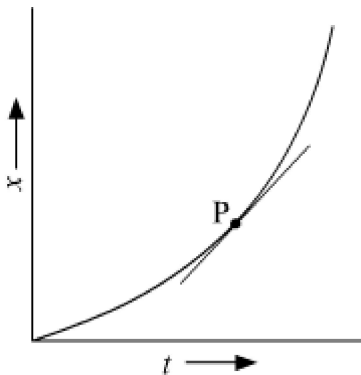
- Average velocity (slope of the $x-t$ graph)



$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

∴ Average velocity = slope of $\overline{P_1P_2}$

- **Average speed** = Total path length / Total time interval
- [No direction is considered]
- **Instantaneous velocity:**



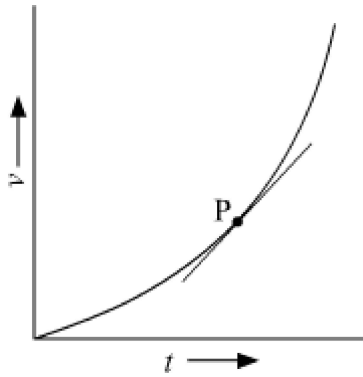
$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

= slope of the tangent at point P

- **Average acceleration:**

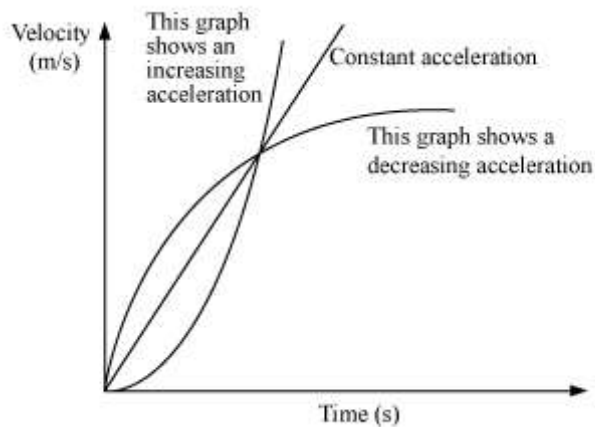
$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

- **Instantaneous acceleration:**

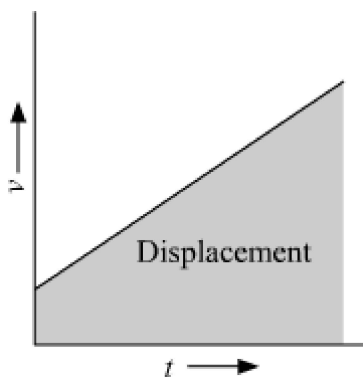


$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \text{slope of the tangent at point P}$$

- Velocity-time graph showing constant acceleration, increasing acceleration and decreasing acceleration:



- Area under the v - t curve is equal to the displacement of the body.



Equations of motions (Kinematic equations) [When acceleration is uniform]

- Velocity-time relation

$$v = u + at$$

- Distance-time relation

$$s = ut + \frac{1}{2}at^2$$

- Velocity-displacement relation

$$v^2 - u^2 = 2as$$

- Distance travelled in n^{th} second of uniformly accelerated motion is given by the relation,

$$D_n = u + \frac{a}{2}(2n-1)$$

Galileo's law of odd number

- The ratios of the distance covered by a body falling from the rest increase by odd numbers from one second to the next. That means, distances covered by each will increase by factors of 1, 3, 5, 7, ...

Relative Velocity

- The relative velocity of a body **A** with respect to another body **B** (v_{AB}) is the time rate at which **A** changes its position with respect to **B**.
 - **Case 1: Both bodies move in the same direction:** If **A** and **B** are moving in the same direction, then the resultant relative velocity is $v_{AB} = v_A - v_B$
 - **Case 2: The bodies move in opposite directions:** If **A** and **B** are moving in the opposite directions, then the resultant relative velocity is $v_{AB} = v_A + v_B$

- $\vec{a} \rightarrow \vec{v}' \rightarrow -\vec{v} \rightarrow \Delta t = \Delta \vec{v} \rightarrow \Delta t$ Displacement vector can be written as

$$\Delta \vec{r} \rightarrow \vec{r}' \rightarrow -\vec{r} \rightarrow x' - x\hat{i} + y' - y\hat{j} + z' - z\hat{k} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

- **Velocity:**

- Average velocity, $\vec{v} \rightarrow \Delta \vec{r} \rightarrow \Delta t$
- Instantaneous velocity, $\vec{v} \rightarrow d\vec{r} \rightarrow dt = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

Where, $v_x = dx/dt$, $v_y = dy/dt$, $v_z = dz/dt$

- **Acceleration:**

- Average acceleration, $\vec{a} \rightarrow \vec{v}' \rightarrow -\vec{v} \rightarrow \Delta t = \Delta \vec{v} \rightarrow \Delta t$
- Instantaneous acceleration, $\vec{a} \rightarrow d\vec{v} \rightarrow dt = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Where, $a_x = dv_x/dt$, $a_y = dv_y/dt$, $a_z = dv_z/dt$

- Motion in a plane can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.



- **Relative Velocity in Two Dimensions:** If two objects A and B are moving with velocities $\vec{v} \rightarrow A$ and $\vec{v} \rightarrow B$.
 - velocity of object A relative to that of B is

$$\vec{v} \rightarrow AB = \vec{v} \rightarrow A - \vec{v} \rightarrow B$$

- velocity of object B relative to that of A is

$$\vec{v} \rightarrow BA = \vec{v} \rightarrow B - \vec{v} \rightarrow A$$

- If the man wants to protect himself from rain, then he should hold an umbrella at an angle $\theta = \tan^{-1} \frac{v_m}{v_r}$ towards his motion with the vertical.
- The motion of a projectile may be thought of as the result of horizontal and vertical components.
- Vertical component comes under accelerated motion because of acceleration due to gravity acting downwards but horizontal component is under uniform motion.
- Both the components act independently.
- **Equation of the path of a projectile:**

$$y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

This is the equation of a parabola. Hence, the path of a projectile is a parabola.

- **Time of flight (T):** It is total time for which an object is in flight.

Total time of flight = Time of ascent + Time of descent

$$T = \frac{2u \sin \theta}{g}$$

- **Maximum height:** Maximum height h reached by the projectile,

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

- **Horizontal Range:** It is the horizontal distance covered by the object between its point of projection and the point of hitting the ground. It is denoted by R .

$$R = \frac{u^2 \sin 2\theta}{g}$$

Horizontal range is maximum if the angle of projection is 45° .

- **Angle of projection:** It is the angle made by velocity of projection with the horizontal.

Angle of projection,

$$\theta = \tan^{-1} \frac{H}{R}$$

Cases :

$$\text{When } \theta_0 = 45^\circ,$$

$$R = u^2/g$$

$$R_{\max} = 4H_{\max}$$

$$\text{When } \theta_0 = 90^\circ,$$

$$H = \frac{u^2}{2g}$$

$$R = 0$$